An Error Model for Quantum Circuits Using Ion Traps

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Outline

- **Motivation**
- **Quantum computing**
- Linear ion traps
- Challenges to implementation
- Previous work
- Proposed error model
- Summary
- Future work

Motivation

- Practical success of quantum computing hinges on error-correcting codes
- Error-correction based on error model
- Standard error model makes many assumptions
- Practical error models derive from technology
- Ion traps among most promising hardware technologies

Motivation

- Detailed physical simulation of noisy phenomena computationally infeasible
- Logic level error model more tractable
  - Stuck-at fault model adequate for VLSI testing
  - Correspondence between logical and physical errors exists for most defects
- Quantum systems
  - Physics-driven simulation: quantum mechanics
  - Is an adequate logic-level characterization of errors possible?

Quantum Computing

- Qubit state
  - $|\Psi\rangle = \alpha|0\rangle + \beta|1\rangle$, $\alpha, \beta \in \mathbb{C}$, $|\alpha|^2 + |\beta|^2 = 1$
  - $|\Psi\rangle_{AB} = |\Psi\rangle_A \otimes |\Psi\rangle_B$
- Quantum gates
  - Unitary operators in $\mathbb{C}^{\otimes n} \times \mathbb{C}^{\otimes n}$
  - Reversible evolution
- Measurements collapse superpositions to an eigenstate of measurement operator

Quantum Computing

- Asymptotic speedup of certain problems
  - Memory grows exponentially with qubits!
  - Entanglement: not observed in classical case
  - Shorís algorithm
    - Number factoring, n bits
    - $O(n^2 \log n \log\log n)$
    - Classical (number-field sieve) $\exp(\Theta(n^{1/3} \log^{2/3} n))$
  - Groverís algorithm
    - Search through unstructured database, size N
    - $\Theta(\sqrt{N})$
    - Classical $O(N)$

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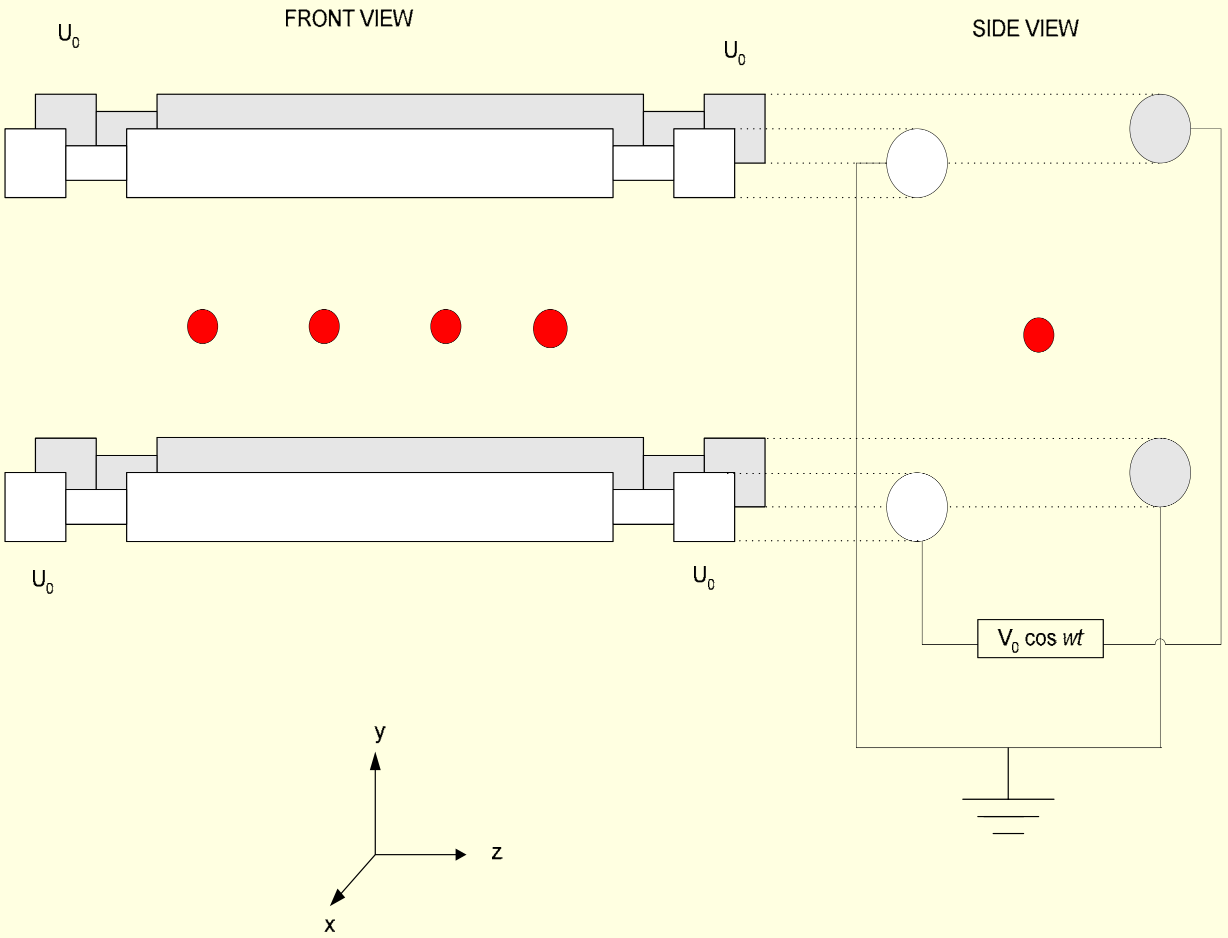
Linear Ion Traps

- Requirements from quantum hardware ♣
  - Robust representation of quantum information
  - Universality of logical transformations
  - Initial state preparation with high fidelity
  - Easy high-performance measurements

Nielsen and Chuang, Chp. 7

Linear Ion Traps



Linear Ion Traps: Apparatus

- Four cylindrical electrodes
  - Two earthed
  - Two connected to sinusoidal potential
    - Radial confinement
  - These also have static potentials at ends
    - Axial confinement of ions ñ balance between Coulombic repulsion from electrode and each other.
  - Radial forces >> axial forces => linear

Linear Ion Traps: Logic

- Quantum states
  - Atomic spin ($^9Be^+$): internal state
  - Axial COM motion: motional state
    - Means of coupling for controlled gates
  - $|\Psi\rangle = \cos\theta\,|0\rangle + e^{i\varphi}.\sin\theta\,|1\rangle$

- Quantum gates
  - Single qubit: zap ion with 1 laser pulse
    - Duration governs θ, phase governs φ
  - Controlled NOT [j,k] : 3 laser pulses
    - Couple spin of ion j with motional mode
    - Transform spin of ion k if phonon in motional mode
    - Decouple spin of ion j from motional mode
  - Universal quantum logic can be implemented

Linear Ion Traps: Logic

- Initial state preparation: laser cooling
  - Doppler cooling: ion-momentum loss to recoil of colliding photon
  - Sideband cooling: photons absorb energy from lower harmonic of fundamental vibration frequency.
- Measurement: light of 313 nm wavelength
  - $|0\rangle$ flouresces due to special transitions
  - $|1\rangle$ remains dark

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Challenges to Implementation

- **Individual Ion Addressing**
  - Ion separation $\propto$ (# ions)$^{-0.56}$
  - Beam focusing
    - Tightly focused beams ñ high transverse gradients => high spatial precision required
    - Focus beam through sharp aperture and image onto ion using lens ñ good gradients
  - Destructive interference of counter-propagating Raman beams.
  - Magnetic field gradients to shift position
  - RF-field induced micromotion

Challenges to Implementation

- **Multimode interference**
  - n qubits cause 3n modes of vibration
    - 1 axial COM mode of interest
    - 3n-1 spectator modes
  1. Spectator-motion effect on logic gates
     - Some operations sensitive to frequency that is a function of all modes of motion
  2. Static electric field imperfections
     - Non-quadratic potentials with jitter causes mode cross-coupling
     - Net gain/loss of energy →redefinition of frequencies

Challenges to Implementation

- **Multimode interference** [contd]
  3. Logic gate induced mode-cross coupling
     - Spectator modes with frequency-sum or difference ~ transition frequency get coupled to transition states.

Challenges to Implementation

- **Decoherence**
  1. Spontaneous emission
  2. Motional decoherence
     - Heating from RF fields in trap
     - Collisions with background atoms
     - Fluctuating electrode potentials
     - Thermal noise from lossy elements in electrodes
     - Electron field emission from electrodes
     - Coupling of moving charge chain with spurious external EM fields

Challenges to Implementation

- **Decoherence** [contd.]
  3. Noise from applied field
     - Inaccuracy in targeting ion
     - Laser pulse timing imprecision
     - Intensity fluctuations

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Previous work

- **Standard error model** [Nielsen & Chuang]
  - Bit flip: $X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$
  - Phase flip: $Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$
  - Bit-phase flip: $Y = \begin{bmatrix} 0 & -\boldsymbol{i} \\ \boldsymbol{i} & 0 \end{bmatrix}$

With I, these form basis for space of 2×2 matrices

Previous Work

- **Continuum of Operational Errors**
  - Obenland and Despain, Univ. S. California
  - Single qubit rotations
    - W(θ,φ) = $$\begin{bmatrix} \cos\frac{\theta}{2} & -\boldsymbol{ie}^{\boldsymbol{i}\varphi}\sin\frac{\theta}{2} \\ -\boldsymbol{ie}^{-\boldsymbol{i}\varphi}\sin\frac{\theta}{2} & \cos\frac{\theta}{2} \end{bmatrix}$$
  - Errors: over-rotations or under-rotations
    - W(θ+Δθ,φ+Δφ)
    - Error angles Δθ, Δφ chosen as per probability distribution
    - Fixed magnitude and sign ñ mis-calibration, bad equipment
    - Fixed magnitude, random sign ñ control imprecision
    - Pseudo-gaussian with given variance ñ random noise phenomena

Previous Work

- Continuum of Operational Errors [contd.]
  - Controlled-NOT implemented as three 1-qubit rotations (third logic level used as intermediate state)
  - Each step accumulates errors

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Proposed Error Model

1. **Over-rotations and under-rotations**

- Due to inaccuracy and imprecision in controlling various parameters of radiation
  - Timing
  - Intensity (jitter, targeting)
- Intended Transformation

$$\boldsymbol{W}(\theta,\varphi)=\begin{bmatrix}\cos\theta & -\boldsymbol{ie}^{i\varphi}\sin\theta\\ -\boldsymbol{ie}^{-i\varphi}\sin\theta & \cos\theta\end{bmatrix}$$

- Resulting transformation

$$\boldsymbol{W}(\theta+\Delta\theta,\varphi+\Delta\varphi)$$

Proposed Error Model

2. Qubit coupling

- Correlated error
- Wide laser pulse can illuminate neighbor ion
- Intended transformation at target ion

$$\boldsymbol{W}(\theta,\varphi)=\begin{bmatrix}\cos\theta & -\boldsymbol{i}\boldsymbol{e}^{\boldsymbol{i}\varphi}\sin\theta \\ -\boldsymbol{i}\boldsymbol{e}^{-\boldsymbol{i}\varphi}\sin\theta & \cos\theta\end{bmatrix}$$

- Transformation at affected neighbor

$$\boldsymbol{W}(\Delta\theta,\Delta\varphi)$$

  - $\Delta\theta$, $\Delta\varphi$ fractions of θ and φ respectively.
- Like coupling faults in semiconductor memories

Proposed Error Model

- **Stuck-at faults**
  - Stray EM fields in environment
  - Interaction with background particles
  - Thermal noise from electrodes
  - RF heating within trap
- Qubit measured in some basis, or
- Qubit behaves like a basis state but isnít in it.

Proposed Error Model

3. ***Benign* stuck-at faults**
   - Measurement by environment
   - Not necessarily in computational basis
   - Logical transformations continue afterwards
   - Not truly stuck-at: qubit not a physical wire
   - Resulting transformation
     - Projective measurement operator
       - Depends on basis
       - Depends on state of qubit (normalization)

Proposed Error Model

4. ***Catastrophic* stuck-at faults**
   - Qubit transition to non-computational-basis state
   - No further reaction to subsequent pulses
     - Invariant to further transformations
   - No flourescence in 313 nm light
     - Measurement yields $|1\rangle$ (qubit remains dark)
   - No coupling of internal and motional states
     - Controlled logic:
       - If control qubit, behaves as classical $|0\rangle$!
       - If target qubit, invariant to transformation

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Summary

- Ion trap quantum computation explored
- Sources of error identified
  - Operational faults
  - Decoherence by environment
- Previously proposed error model extended
  - Qubit coupling
  - Measurement
  - Depolarization

Future Work

- Modeling spectator mode coupling effect
    - Characterized by an interaction Hamiltonian
- Simulations of quantum algorithms
    - This error model to be used
    - Comparison with standard error model
    - Effectiveness of known error-correcting codes
- Implications to error-detection and correction