We propose a nearly optimal algorithm that uses $2n - 2$ random parameters, $O(n)$ memory space and $O((n \log 2n) \log \log n)$ operations in a fixed arbitrary field in order to compute the rank and basis for the null space of a structured $n \times n$ matrix $X$ represented with $O(n)$ parameters of its short generator, as well as to solve a linear system $Xy = b$ or to determine its in-consistency. If rank $X = n$, the algorithm also computes $\det X$ and a short generator of $X - I$. The cost bounds cover correctness verification for the output but not the cost of the generation of random parameters. The algorithm gives a unified treatment of various classes of structured matrices including ones of Toeplitz, Hankel, Vandermonde and Cauchy types.